

# A Control System Framework for Privacy Preserving Demand Response of Thermal Inertial Loads

Abhishek Halder, Xinbo Geng, Gaurav Sharma, Le Xie, and P. R. Kumar  
 Department of Electrical and Computer Engineering, Texas A&M University  
 {ahalder, gengx@tamu, gash, le.xie, prk}@tamu.edu

**Abstract**—We address a problem in demand response for the class of thermal inertial loads: How to control the total demand of a set of loads, and also what to control it to, all while respecting constraints on load states, as well as maintaining privacy of load states. The architecture we investigate consists of an aggregator or Load Serving Entity (LSE) serving several thermostatically controlled loads (TCLs), whose set-points are controlled by the LSE to follow a desired total power consumption. We propose a modeling, analysis and design framework for the LSE that accomplishes this goal by solving a hierarchy of two control problems. The first control problem consists of determining an optimal aggregate power trajectory given the price forecasts. The second problem is that of controlling the set-points of the TCL population so as to drive their total power consumption to follow the optimal power trajectory. We show how the LSE can solve these control problems while guaranteeing privacy and contractual comfort bounds for the TCL states. Numerical simulations illustrate the efficacy of the proposed framework.

## I. INTRODUCTION

Demand response [1] refers to adjusting the energy consumed by loads to partially balance supply side variability. It is expected to play an important role in increasing utilizability of renewable energy sources. The objective of this paper is to propose a holistic framework for demand response for the class of thermostatically controlled loads (TCLs).

TCLs, such as air conditioners, refrigerators and swimming pool pumps, constitute almost 50% of the total energy consumption in United States [2], and, due to their thermal inertia, are well suited [3] to provide flexibility for the grid. In this setting (see Fig. 1), an independent system operator (ISO) requests the aggregator or Load Serving Entity (LSE) to control a large population of TCLs over some time interval, so as to meet a prescribed total energy consumption. The LSE, then faces the twin problems of determining the level of demand response commitment to the ISO, as well as the problem of achieving this level through load aggregation.

This architecture raises several fundamental questions:

- How should the loads be controlled so that the total power consumed by the collection of loads follows a given trajectory over time, while still respecting each TCL's constraints on its maximum and minimum temperature experienced?
- How does one achieve this demand response while still respecting the privacy [4] of individual loads?

This material is based upon work partially supported by NSF under Contract Nos. ECCS-1546682, CPS-1239116, Science & Technology Center Grant CCF-0939370, DGE-1303378, and ECCS-1150944.

- If the energy price paid by the LSE varies over time, how should it determine the minimum cost trajectory of total power consumption by the collection of all TCLs?
- Suppose there are several classes of loads, with each class having a specified temperature range for its comfort, codified in a contract with the LSE. What is the cost of such a contract for the LSE?

We propose a framework for modeling, analysis and design of such an LSE-based demand response system serving a large collection of TCLs.

### A. Related Work and Contributions of This Paper

There has been a large body of literature [5]–[11] investigating different aspects of controlling TCLs to achieve demand response. Policies such as set-point displacement control [5]–[9], switching rate control [10], and set-point velocity control [11] have been considered.

We propose and investigate a hierarchical control framework (see Fig. 2). At the first layer, we show how to formulate the problem of determining the aggregate power trajectory under real-time pricing as an optimal control problem, so that the LSE can minimize its total cost of energy procurement while satisfying the consumers' contractual obligations. To the best of the authors' knowledge, formulating the design of reference power trajectory for demand response as a control problem, while ensuring end users' privacy, seems novel with respect to the existing literature. At the second layer, we show how a simple PID velocity controller for set-point can be used by the LSE to track the optimal aggregate power trajectory while respecting consumers' privacy and contracts. Numerical simulations illustrate the trade-off between contractual obligations and minimizing the cost of energy consumption.

## II. THE MODEL

In this paper we assume the TCLs are cooling houses (e.g., in Texas summer). Focusing on a particular TCL, let us denote its temperature at time  $t$  by  $\theta(t)$ , and the ambient temperature of the outside environment by  $\theta_a(t)$ . Left uncooled, the temperature of the TCL will exponentially rise towards  $\theta_a$ . However, if cooled by an air conditioner using power  $P$  drawn from the grid, the temperature decreases, let us suppose, by an additional rate  $\beta P$ . Let  $\sigma(t)$  be a 0/1 valued indicator function that indicates whether the air conditioner is OFF/ON. Then the temperature of the TCL follows the differential equation

$$\dot{\theta}(t) = -\alpha(\theta(t) - \theta_a(t)) - \beta P \sigma(t).$$

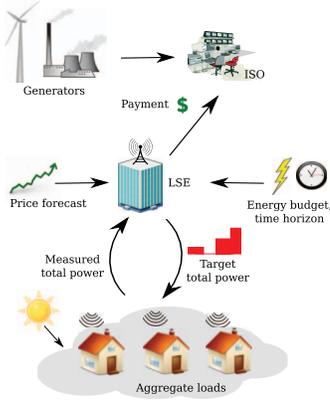


Fig. 1: Architecture of the proposed demand response system.

The parameters  $\alpha, \beta > 0$  denote the heating time constant and thermal conductivity, respectively. We will suppose, only for simplicity of notation and exposition, that the TCL population is homogeneous, with identical values of  $\alpha, \beta, P$ .

We suppose that each TCL has a comfort zone  $[L_0, U_0] \triangleq [s_0 - \Delta, s_0 + \Delta]$  within which its temperature is required to lie at all times. If  $\Delta$  is large (small), then the TCL has more (less) flexibility about its temperature, and hence more (less) thermal inertia that the LSE can exploit to time when the TCL is ON and when it is OFF, while still keeping that TCL's temperature in its comfort zone. This allows the LSE to postpone power purchase when the price of power is expected to be high, and to purchase it when the price is low, and store it as thermal inertia.

We assume that each TCL is allowed to choose its  $\Delta$  from a fixed interval  $[\Delta_{\min}, \Delta_{\max}]$ . Such a choice will constitute a "contract" between the TCL and the LSE. The LSE guarantees that during demand response, the temperature  $\theta(t)$  of the TCL will lie within its specified comfort zone, i.e., it will satisfy

$$L_0 \leq \theta(t) \leq U_0 \Leftrightarrow s_0 - \Delta \leq \theta(t) \leq s_0 + \Delta, \text{ for all } t. \quad (1)$$

We assume that the initial temperature  $\theta(0)$  satisfies (1).

Let us suppose that at each time  $t$ , the TCL has a "upper set-point"  $U_t \triangleq s(t) + \Delta$ , and a "lower set-point"  $L_t \triangleq s(t) - \Delta$ . Since the set-point  $s$  may change over time, the variable governing its change will comprise the LSE's "control" variable.

The TCL turns itself ON when the temperature hits  $U_t$ , and turns itself OFF when the temperature hits  $L_t$ . The functioning of the air-conditioner is hysteretic, i.e., if it is ON (OFF), then it remains ON (OFF) until it hits the lower (upper) set-point. The "switching trajectory"  $\sigma(t)$  satisfies

$$\sigma(t) = \begin{cases} 1 & \text{if } \theta(t) \geq U_t, \\ 0 & \text{if } \theta(t) \leq L_t, \\ \sigma(t^-) & \text{otherwise.} \end{cases}$$

The state of a TCL is  $\{s(t), \theta(t), \sigma(t)\} \in \mathbb{R}^2 \times \{0, 1\}$ .

We will consider an architecture where the LSE changes the set-point  $s(t)$  by issuing a "control signal" (or broadcasting a command signal)  $v(t)$ . Specifically, the LSE requests that

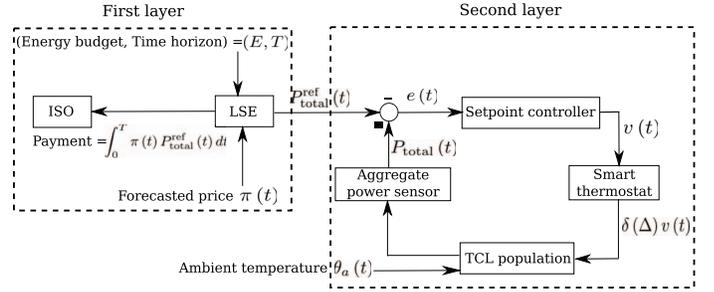


Fig. 2: Block diagram for the hierarchical control framework.

each TCL changes its set-point according to the differential equation

$$\dot{s}(t) = \delta(\Delta) v(t). \quad (2)$$

We envisage that different TCLs will have different comfort ranges entailing different  $\Delta$ s, and so the rate of change of the set-point depends on  $\Delta$  through  $\delta(\cdot)$ .

We do not require the LSE to know the set-point  $s(t)$  of an individual TCL. Rather the LSE only broadcasts the command  $v(t)$  to all the TCLs that it serves and leaves it to each TCL to implement its rate of change  $\delta(\Delta) v(t)$  of its own set-point  $s(t)$ . Similarly, the LSE does not need to know the temperature  $\theta(t)$  and indicator variable  $\sigma(t)$  of the TCL. Thus, the privacy of each TCL's state is preserved.

We assume that the LSE needs to pay for the total power consumed by all the TCLs. The LSE is exposed to the real-time price, denoted by  $\pi(t)$ . By changing the number of TCLs that are ON, the LSE can change its cost. However, it is required to satisfy (1) for all the TCLs without measuring their states. This constrains the freedom of the LSE in timing its power purchases. Clearly, the cost of fulfilling a TCL's contract is lower when  $\Delta$  is larger.

### III. CONTROL STRATEGY OF THE LSE: SET-POINT CONTROL TO TRACK A TOTAL POWER TARGET

The overall strategy we consider is the following. Based on  $\pi(t)$ , the LSE determines a target total power consumption at time  $t$ , denoted by  $P_{\text{total}}^{\text{ref}}(t)$ . It then chooses the "control signal"  $v(t)$  that it issues to the TCLs so that the resulting total power consumed by all the TCLs is  $P_{\text{total}}^{\text{ref}}(t)$ . We call this a "control strategy of the LSE".

There are two central problems that arise:

- How does the LSE determine the target total power consumption  $P_{\text{total}}^{\text{ref}}(t)$ ?
- How does the LSE choose  $v(t)$  so that the resulting total power consumed by the TCLs is indeed  $P_{\text{total}}^{\text{ref}}(t)$ ?

We will show how one can formulate and solve these two problems as control problems. A schematic of the proposed demand response system is shown in Fig. 2.

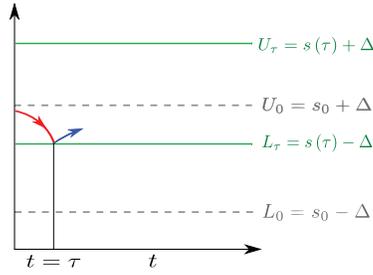


Fig. 3: This figure shows the effect of the same controller (2) with and without temperature constraints (1). Suppose that, during  $[0, \tau]$  the controller increases its set-point to reduce power consumption, resulting in first ON $\rightarrow$ OFF transition at  $t = \tau$ . The  $\theta(t)$  trajectory (in red for ON, blue for OFF) is shown for a TCL that was ON at  $t = 0$ . If the controller ignores constraint (1), then at  $t = \tau$ , the initial dead-band  $[L_0, U_0]$  (dashed grey lines) changes to a new dead-band  $[L_\tau, U_\tau]$  (solid green lines). However, with constraint (1) active, the new dead-band at  $t = \tau$  is  $[L_\tau, U_0]$  (between the upper dashed grey line and lower solid green line).

### A. Designing the Target Total Power Trajectory

We first consider the operational planning problem where the LSE has a forecast of the real-time price  $\pi(t)$  over a time interval  $[0, T]$ . We also allow for the possibility that the LSE is assigned a budget  $E$  for the total energy that it is allowed to consume over the time window  $[0, T]$ .

Suppose that  $u_i(t)$  is a variable that is 1/0, depending on whether the  $i^{\text{th}}$  TCL is ON/OFF. In order to determine the outer limit on what is feasible, we consider the case where each  $u_i(t)$  is directly controlled. Noting that the total power consumed by  $N$  TCLs at time  $t$  is  $P \sum_{i=1}^N u_i(t)$ , and that the total cost of energy over the time interval  $[0, T]$  is  $\int_0^T P_{\text{total}}^{\text{ref}}(t) \pi(t) dt$ , the LSE's objective is to

$$\underset{u_1(\cdot), \dots, u_N(\cdot)}{\text{minimize}} P \int_0^T \pi(t) (u_1(t) + \dots + u_N(t)) dt, \quad (3)$$

subject to

$$\dot{\theta}_i(t) = -\alpha(\theta_i(t) - \theta_a(t)) - \beta P u_i(t), \quad i = 1, \dots, N, \quad (4)$$

$$u_i(t) \in \{0, 1\}, \quad i = 1, \dots, N, \quad (5)$$

$$P \int_0^T (u_1(t) + \dots + u_N(t)) dt = E, \quad (6)$$

$$s_{0i} - \Delta_i \leq \theta_i(t) \leq s_{0i} + \Delta_i, \quad i = 1, \dots, N. \quad (7)$$

The above is an optimal control problem. It can also be formulated and solved in discrete-time, as we do for numerical simulation in Section IV. The time varying Lagrange multipliers (a.k.a. shadow prices) corresponding to the constraint (7) for a specific  $\Delta_i$ , are the marginal cost for the contractual value of  $\Delta_i$ , and thus provide information on how the LSE can price the contract for the  $i^{\text{th}}$  TCL.

### B. Controlling the Set-points of the TCLs

1) *Control law for  $v(t)$* : The next problem is to design the controller (2) for choosing  $v(t)$ , so as to make the total

power consumed by the TCLs indeed track  $P_{\text{total}}^{\text{ref}}(t)$ . This is a standard tracking problem in control. Suppose that the actual power drawn by the set of TCLs,  $P_{\text{total}}(t)$ , can be measured without revealing the power draws of the individual TCLs. Then, one can simply employ a PID controller  $v(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{de}{dt}$ , where the error signal is  $e(t) \triangleq P_{\text{total}}(t) - P_{\text{total}}^{\text{ref}}(t)$ , and  $k_p, k_i, k_d$  are the proportional, integral and derivative gains, respectively. As a model-free strategy that only involves tuning of the gains, the PID controller is easy to implement. (Alternatively, one could design a model-based state-feedback controller, but then one has to address the issue of measuring/estimating the proportion of ON TCLs at time  $t$ , and so we eschew that approach).

2) *Choice of  $\delta(\Delta)$* : The set-point of a TCL with large (small)  $\Delta$  can be moved at a faster (slower) velocity. This motivates choosing  $\delta(\cdot)$  in (2) as an increasing function of  $\Delta$ . In this paper, we choose  $\delta(\Delta) = \Delta$  in the numerical simulations.

3) *Performance*: In the absence of the constraints (1), equation (2) results in  $[L_t, U_t] = [s(t) - \Delta, s(t) + \Delta]$  (see Fig. 3). However, (1) and (2) together result in  $[L_t, U_t] = [L_0 \vee (s(t) - \Delta), U_0 \wedge (s(t) + \Delta)]$ , which, in effect, is equivalent to moving one dead-band boundary, holding the other fixed. Here,  $\vee$  and  $\wedge$  denote maximum and minimum, respectively. Thus, the dead-band width contracts (expands) when the aggregate power consumption needs to be reduced (increased).

## IV. NUMERICAL RESULTS

In this Section, we illustrate our framework on a simple example, and subsequently describe a more realistic case study with actual price ( $\pi(t)$ ) and weather ( $\theta_a(t)$ ) data for Houston, Texas.

### A. Parameters for Numerical Simulation

For each TCL, we employ the following parameter values from Table 1 (p. 1392) in [5]: thermal resistance  $R = 2^\circ\text{C/kW}$ , thermal capacitance  $C = 10 \text{ kWh}/^\circ\text{C}$ ,  $\alpha = \frac{1}{RC} \text{ h}^{-1}$ ,  $\beta = \frac{1}{C} \text{ }^\circ\text{C/kWh}$ , and electrical power drawn  $P = 5.6 \text{ kW}$ . We fix the number of TCLs at  $N = 100$ , and a time horizon of length  $T = 2$  hours. To investigate the effect of contract distribution, we take  $\Delta_{\min} = 0.1^\circ\text{C}$  and  $\Delta_{\max} = 1.1^\circ\text{C}$ , and then consider four scenarios: (i) the entire TCL population has the same  $\Delta \equiv 0.5^\circ\text{C}$ , (ii) each TCL randomly draws  $\Delta$  independently from the uniform distribution over  $[\Delta_{\min}, \Delta_{\max}]$ , (iii) each TCL randomly draws  $\Delta$  independently from a right triangular distribution over  $[\Delta_{\min}, \Delta_{\max}]$  with peak at  $\Delta_{\max}$ , (iv) same as (iii) but with peak at  $\Delta_{\min}$ . In the third (fourth) scenario, there are more consumers with large (small)  $\Delta$ .

The initial conditions  $(s_0, \theta_0)$  for the TCL population are assumed to have a bivariate Gaussian distribution  $\mathcal{N}(\mu_0, \Sigma_0)$ , with  $\mu_0 = (20, 20)^\top$ , and  $\Sigma_0 = \begin{bmatrix} 1 & 0.5 \\ 0.5 & 3 \end{bmatrix}$ . Given the contract ( $\Delta$ ) distributions as in scenarios (i)-(iv), we generate  $N = 100$  random pairs  $(s_0, \theta_0)$  from the Gaussian distribution subject to the constraint  $s_0 + \Delta \leq \theta_0 \leq s_0 + \Delta$ . Then, with probability 0.5, we assign  $\sigma_0 = 1$  or 0. The resulting initial

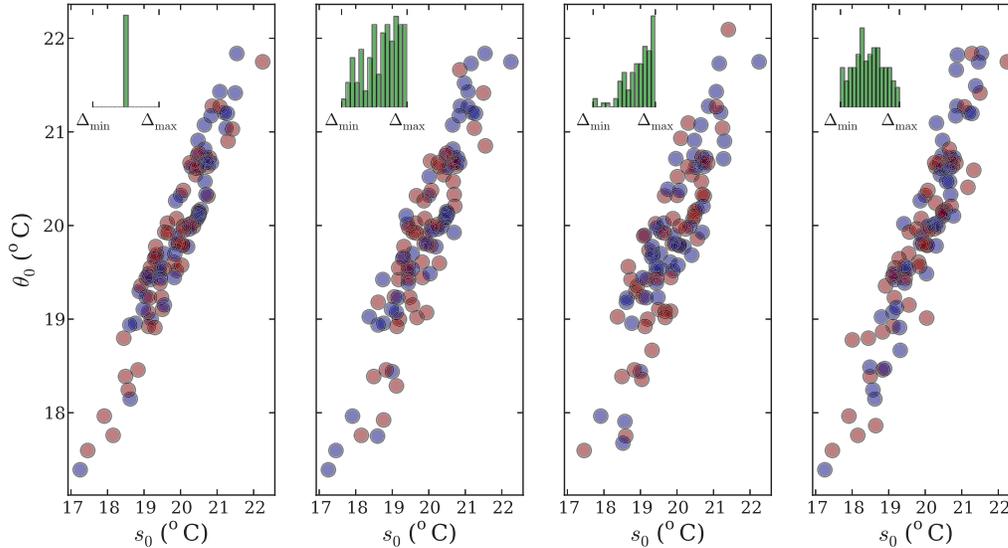


Fig. 4: The plot shows  $N = 100$  random samples drawn from the initial TCL state distribution, for each of the four different contract scenarios (i)-(iv) given in Section IV.A, from *left to right*. The inset histogram in each plot shows the corresponding contract ( $\Delta$ ) distribution for the TCL population. Here, *red* denotes  $\sigma_0 = 1$  (ON) and *blue* denotes  $\sigma_0 = 0$  (OFF).

samples are shown in Fig. 4. These values for TCL parameters, contracts and initial conditions are used throughout the following Sections IV-B and IV-C.

### B. Illustratory Example

1) *Designing  $P_{\text{total}}^{\text{ref}}(t)$* : We compute  $P_{\text{total}}^{\text{ref}}(t)$  for the given real time price forecast  $\pi(t)$  and ambient temperature  $\theta_a(t)$  data shown in Fig. 5. The two-hour data in Fig. 5 is for every 5 minutes.

To numerically solve (3)–(7), we consider the corresponding time-discretized optimization problem. Since  $\theta(t)$  is a real variable but  $u(t)$  is binary, this leads to a mixed-integer linear program. However, since the real-time price is available every 5 minutes, the controller has a time resolution (0.01 seconds) that is much shorter than the time resolution (300 seconds) for designing the optimal reference power trajectory. As a result,  $u(t)$  in our discretized problem, actually represents the *average* state of each TCL over 300 seconds duration, and hence allows the relaxation  $u(t) \in [0, 1]$ , in lieu of  $u(t) \in \{0, 1\}$ . This helps us to solve (3)–(7) as a linear programming problem, whose results are shown in Fig. 7.

The four columns in Fig. 7, from left to right, correspond to contract scenarios (i)-(iv), respectively. From Fig. 7a, it can be seen that for all contractual distributions, the TCLs are actively pre-cooled before the peak price period, thus avoiding power usage during the peak price hours.

- *Not every TCL matters*: In Figs. 7b and 7c, only a small fraction of the TCLs have positive shadow prices.
- *Pre-cooling and after-heating*: The shadow prices  $\lambda_{\text{lower}}(t)$  are positive just before the peak price period, indicating that the TCLs are *pre-cooling*. Similarly, the

shadow prices  $\lambda_{\text{upper}}(t)$  are positive after the peak price period, indicating *after-heating*.

2) *Controlling TCLs*: We discretize the closed-loop TCL dynamics using the Euler method with step-size  $dt = 0.01$  seconds. For proof-of-concept demonstration, we choose constant  $P_{\text{total}}^{\text{ref}}(t) \equiv 50P = 50\%$  of  $NP$ , and  $\theta_a(t) \equiv 32^\circ\text{C}$ .

In Fig. 8, we show the tracking performance of the controller. The four columns in Fig. 8 correspond to the different contract scenarios (i)-(iv). The *top* (*bottom*) row corresponds to the case where the controller *does not* (*does*) respect the contractual obligation (7). Both rows indicate that positive gains  $k_p$  and  $k_d$  together with gain  $k_i = 0$  perform the best, slightly better than  $k_p$  alone. Positive  $k_i$  seems to drive the power curve to zero, while  $k_d$  alone leads to overshoot. All these conform well to traditional PID control behavior, if we keep in mind that ours is “velocity” control, and hence  $k_p$  is really the integral gain,  $k_d$  plays the role of proportional gain, and  $k_i$  is the *double integral* gain.

If we compare the results between different columns, then for the first row, one can see that the identical  $\Delta$  case (scenario (i), first column), and the case when the population has more small  $\Delta$  TCLs (scenario (iv), last column), have large tracking errors, whereas the case when the population has more large  $\Delta$  TCLs (scenario (iii), third column) results in smaller tracking errors at each time, for all PID gain combinations, which matches well with intuition. Similar trends can be observed in the second row, regarding the effect of contract distribution. More interestingly, the plots in the second row of Fig. 8 confirm our reasoning (Fig. 3) that good tracking performance is possible as long as not all TCL dead-bands are squeezed to zero effective width. If that happens, as it does in the first

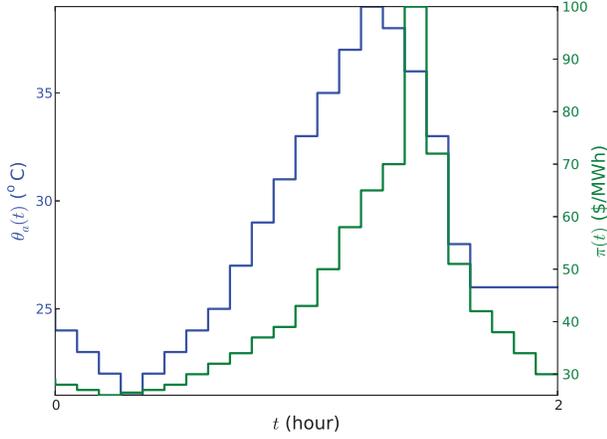


Fig. 5: Real time price forecast  $\pi(t)$  (green) and ambient temperature  $\theta_a(t)$  (blue) for Section IV.B.1.

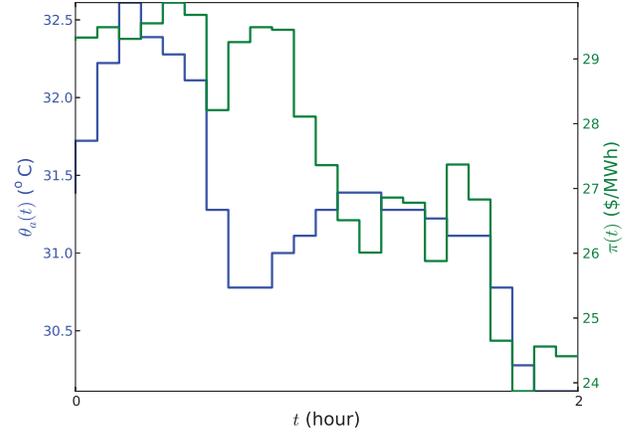
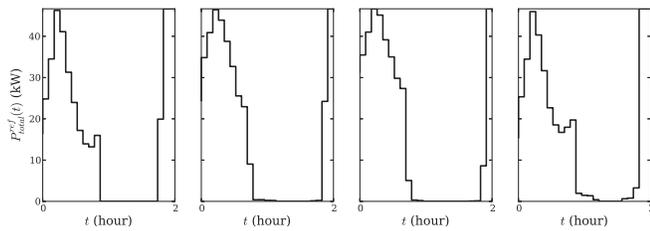
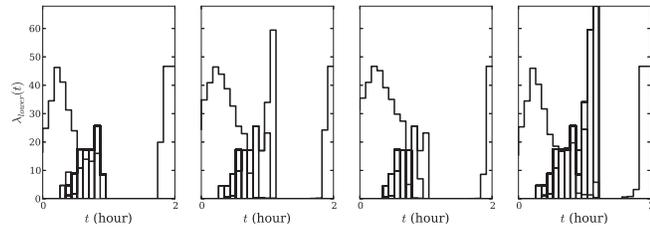


Fig. 6: Real time price forecast  $\pi(t)$  (green) and ambient temperature  $\theta_a(t)$  (blue) for Section IV.C.

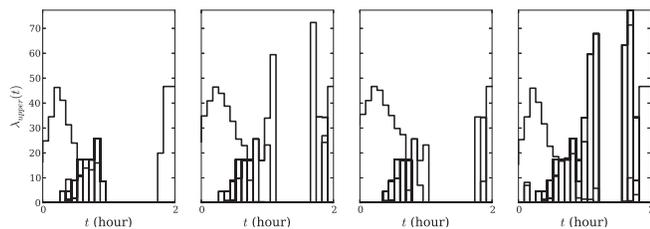
and fourth column of the second row in Fig. 8, the number of switchings increases with time and  $P_{\text{total}}(t)$  leaves the tracked reference, and in the long run, converges to the uncontrolled aggregate power with increasing frequency.



(a) Reference total power trajectory  $P_{\text{total}}^{\text{ref}}(t)$



(b) Shadow prices  $\lambda_{\text{lower}}(t)$  for the lower bound in (7)



(c) Shadow prices  $\lambda_{\text{upper}}(t)$  for the upper bound in (7)

Fig. 7: Optimal solution for the numerical example in Section IV.B.1 corresponding to the  $\pi(t)$  and  $\theta_a(t)$  data given in Fig. 5. Different columns (from left to right) correspond to the different contract scenarios (i)-(iv).

### C. Case Study

We now consider the actual price [12] and ambient temperature [13] data for Houston, for May 20, 2015, 4–6pm (Fig. 6). These data files can be downloaded from [14]. For contract scenarios (i)-(iv), the optimal solutions  $P_{\text{total}}^{\text{ref}}(t)$ , are shown as the *solid black lines* in Fig. 9, and have trends similar to those observed in Fig. 7a. Next, from Fig. 9, we observe trends similar to those observed in Fig. 8 that, without contractual obligation (first row of Fig. 9), the controller has satisfactory tracking performance. As seen in the second row of Fig. 9, for a moderate duration, the controller can achieve good tracking performance while meeting contractual constraints. However, if required to reduce power for prolonged duration, the tracking performance cannot be met without violating the consumers' contractual obligations.

## V. CONCLUSIONS

We have proposed a modeling, analysis and design framework for demand response, employing an aggregator or Load Serving Entity (LSE), serving a population of thermostatically controlled loads (TCLs). We have shown that the demand response problem for the LSE can be decomposed into two distinct problems, the problem of determining a demand response given the real-time price forecast, and the problem of changing the set-points of the TCL populations in a way to realize the desired demand response. The first is an optimal control problem that can be solved numerically. The second is a tracking problem that can be solved by a simple model-free PID controller. The comfort range constraint of each TCL is codified as a contract between the LSE and the TCL. We have proposed a distributed contract-aware implementation for TCL set-point control. Our solution respects both the comfort range constraints specified by each TCL, as well the privacy of its state. The comfort range constraint limits the amount of aggregate thermal inertia that the LSE can exploit. We have

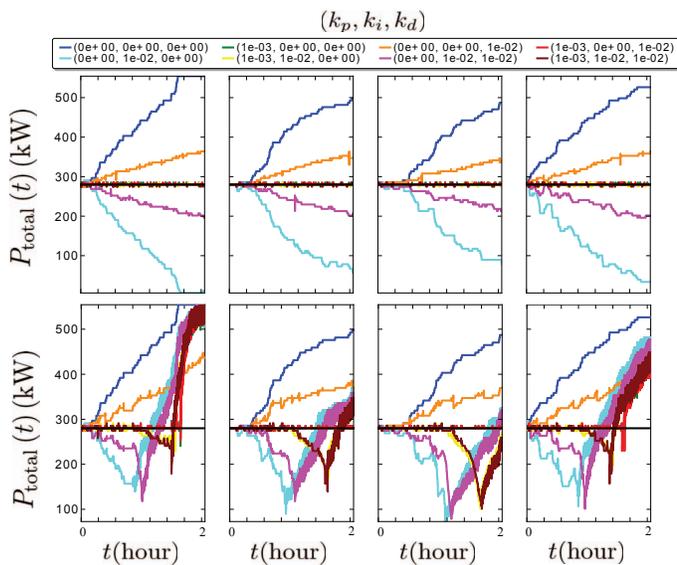


Fig. 8: Aggregate output power  $P_{\text{total}}(t)$  is shown for the model in Section IV.B.2, where the controller (2) has the objective of tracking a constant  $P_{\text{total}}^{\text{ref}}$  (solid black line). The columns (from left to right) correspond to the contract scenarios (i)-(iv), respectively, while the rows correspond to with (top row) and without (bottom row) contractual obligation. The line colors in each plot correspond to different PID gain tuples shown in the legend.

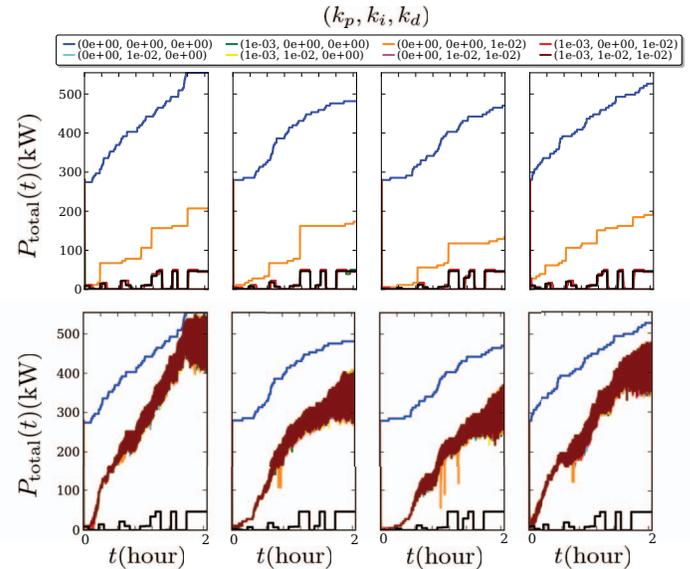


Fig. 9: The total power consumed,  $P_{\text{total}}(t)$ , is shown for the case study in Section IV.C, where the controller (2) has the objective of tracking the designed  $P_{\text{total}}^{\text{ref}}$  (solid black line). The columns (from left to right) correspond to the contract scenarios (i)-(iv), respectively, while the rows correspond to with (top row) and without (bottom row) contractual obligation. The line colors in each plot correspond to different PID gain tuples shown in the legend.

illustrated our framework on a case study with actual price forecast and ambient temperature data for Houston area.

## REFERENCES

- [1] U.S. Department of Energy, "Benefits of Demand Response in Electricity Markets and Recommendations for Achieving Them". A Report to the United States Congress Pursuant to Section 1252 of the Energy Policy Act of 2005, Feb. 2006.
- [2] Energy Information Administration, "Residential Energy Consumption Survey". U.S. Department of Energy, Washington D.C., Tech. Report, 2001.
- [3] D.S. Callaway, and I.A. Hiskens, "Achieving Controllability of Electric Loads". *Proceedings of the IEEE*, Vol. 99, No. 1, pp. 184–199, 2011.
- [4] M.A. Lisovich, D.K. Mulligan, and S.B. Wicker, "Inferring Personal Information from Demand-Response Systems". *IEEE Security & Privacy*, Vol. 8, No. 1, pp. 11–20, 2010.
- [5] D.S. Callaway, "Tapping the Energy Storage Potential in Electric Loads to Deliver Load Following and Regulation, with Application to Wind Energy". *Energy Conversion and Management*, Vol. 50, No. 5, pp. 1389–1400, 2009.
- [6] S. Kundu, N. Sinitsyn, S. Backhaus, and I.A. Hiskens, "Modeling and Control of Thermostatically Controlled Loads". *Proceedings of the 17th Power Systems Computation Conference*, Stockholm, Sweden, 2011.
- [7] S. Bashash, and H.K. Fathy, "Modeling and Control of Aggregate Air Conditioning Loads for Robust Renewable Power Management", *IEEE Transactions on Control Systems Technology*, Vol. 21, No. 4, pp. 1318–1327, 2013.
- [8] G. Sharma, L. Xie, and P.R. Kumar, "Large Population Optimal Demand Response for Thermostatically Controlled Inertial Loads". *Proceedings of the IEEE International Conference on Smart Grid Communications (SmartGridComm)*, pp. 259–264, 2013.
- [9] G. Sharma, L. Xie, and P.R. Kumar, "Optimal Demand Response for Thermal Inertial Loads Employing Stochastic Renewables: A Privacy Respecting Architecture and its Continuum Scaling Limit". *Proceedings of the 53rd IEEE Conference on Decision and Control*, pp. 358–363, 2014.
- [10] L.C. Totu, and R. Wisniewski, "Demand Response of Thermostatic Loads by Optimized Switching-Fraction Broadcast". *2014 IFAC World Congress*, Vol. 19, No. 1, pp. 9956–9961, 2014.
- [11] A. Ghaffari, S. Moura, and M. Krstić, "Analytic Modeling And Integral Control Of Heterogeneous Thermostatically Controlled Load Populations". *Proceedings of the ASME 2014 Dynamic Systems and Control Conference*, pp. V002T22A002–V002T22A002, 2014.
- [12] <http://www.ercot.com/mktinfo/rtm/index>
- [13] <http://tinyurl.com/HoustonDataMay20-4to6pm>
- [14] <http://people.tamu.edu/~ahalder/sgc2015>