Chance Constrained Optimal Reactive Power Dispatch

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***Abstract*—The uncertainties from deepening penetration of renewable energy resources have already shown to impact not only the market operations, but also the physical operations in large power systems. It is demonstrated that deterministic modeling of wind would lead to voltage insecurity in the reality where wind fluctuates. This could render deterministic control of reactive power ineffective. As an alternative, we propose a chance-constrained formulation of optimal reactive power dispatch which considers the uncertainties from both renewables and contingencies. This formulation of a chance constrained optimal reactive power dispatch (cc-ORPD) offers system operators an effective tool to schedule voltage support devices such that the system voltage security can be ensured with quantifiable level of risk. The cc-ORPD problem is a Mixed-Integer Non-Linear Programming (MINLP) problem with a joint chance constraint and is extremely challenging to solve. Using the Big-M approach and linearized power flow equations, the original cc-ORPD problem is approximated as a Mixed-Integer Linear Programming (MILP) problem, which is efficiently solvable. Case studies are conducted on a modified IEEE 24-bus system to investigate the optimal operating schedule under uncertainties and the out-of-sample violation probability.**

I. Introduction

A. Background

The high variability and limited predictability of renewables impose new challenges on the secure and reliable operation of power systems. There has been a substantial amount of literatures showing that deep penetration of renewables could jeopardize the security and reliability of power systems [1]–[3]. For example, the rapid increase and stochastic nature of renewables might lead to voltage issues, which could be severe when a stressed system is lack of reactive support. An Optimal Reactive Power Dispatch (ORPD) problem is often formulated for better voltage profiles [1]–[3]. The ORPD problem aims at finding optimal settings of current installed Reactive Power Support Devices (RPSDs) such as SVCs and Capacitor Banks to ensure system voltage constraints [4]. Although numerous papers have studied the ORPD problem, most of them adopt a deterministic formulation and uncertainties from wind are ignored.

In this paper, we propose a framework for optimal reactive power dispatch considering joint uncertainties from wind and contingencies. The proposed framework is built upon chance-constrained programming, which is a natural and efficient tool for decision making in an uncertain environment.

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B. Chance Constrained Programming

Problem (1) is the typical form of a single-stage chance-constrained program (CCP):

$$\min_{x} \quad c^{\intercal} x \tag{1a}$$

$$\text{s.t.} \quad Ax \geq b \tag{1b}$$

$$\mathbb{P}_{\omega}\Big(G(\omega)x \leq h(\omega)\Big) \geq 1-\epsilon \tag{1c}$$

$$x \in \mathbb{R}^n$$

Problem (1) aims at finding a cost-minimizing strategy while satisfying a set of deterministic and probabilistic constraints. Without loss of generality [5], we assume the objective takes linear form $c^{\intercal}x$. Decision variables are denoted by x, and Eqn. (1b) is the *deterministic* constraint on x. Uncertainties appear as variable $\omega \in \mathbb{R}^m$, and the chance constraint Eqn. (1c) requires the inner constraint $G(\omega)x \leq h(\omega)$ to be satisfied with probability at least $1-\epsilon$.

CCPs are often challenging to solve for the following two reasons: (1) the feasible region of a CCP is usually non-convex [6]; and (2) it is NP-hard to accurately calculate the probability in the chance-constraint [7]. There are four typical methods to get approximately optimal solutions to CCPs: (1) deriving a *deterministic equivalent* optimization problem [8], [9]; (2) *convex approximation* [6]; (3) *scenario approach* [5]; and (4) *Big-M approach* [10]–[12]. Because the cc-ORPD problem is a MINLP problem, the Big-M approach, which is a favorable choice to handle integer variables in CCPs, is selected to solve cc-ORPD in this paper. More details on the Big-M approach is provided in Section III.

C. Chance-constrained Programs in Power Systems

There are many applications of CCPs on power system problems: chance-constrained DCOPF (cc-DCOPF) [13]–[17], chance-constrained Unit Commitment (cc-UC) [18], [19], using chance-constrained programming to handle contingencies in power systems [20], [21]. In this paper, we formulate a chance-constrained Optimal Reactive Power Dispatch (cc-ORPD) problem to address the voltage security issue induced by the deep penetration of renewables and potential contingencies. The cc-ORPD problem is unique in the following three aspects: (1) It is built upon a more accurate model of power system (i.e. AC power flow) rather than the simplified DC power flow model, which appears in most of literatures

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[13], [14], [18]–[20]. (2) The cc-ORPD problem considers the optimal operation of both continuous and discrete state voltage support devices. While in [22], only continuous-state devices (e.g. SVCs) are being considered. (3) The cc-ORPD problem ensures voltage security with respect to the joint distribution of contingencies and wind uncertainties. Whereas most literatures handling contingencies via CCPs [19]–[21] are based on DC power flow model. As a result, they are fundamentally incapable of addressing voltage-related issues.

The remainder of this paper is organized as follows: Section II discusses the impacts of wind uncertainties on voltage security. Section III introduces the Big-M approach to solve CCPs. Motivated by the discussion in Section II, we formulate a cc-ORPD problem in Section IV. Section IV also elaborates how to derive a computationally tractable form of the cc-ORPD problem via the Big-M approach. Case studies and concluding remarks are presented in Section V and Section VI, respectively.

II. Impacts of Wind Uncertainties on Voltage Security

A. Wind Farm Modeling

The wind farm is often modeled as a negative real load or pure real power generator in most literatures. While at most Independent System Operators (ISOs) in the US, wind farms are required to provide some reactive support to reduce voltage issues. In this paper, the wind farm is modeled as a negative load with constant power factor 0.95. Let $P_W \in \mathbb{R}^{|\mathcal{W}|}$ and $Q_W \in \mathbb{R}^{|\mathcal{W}|}$ denote the forecast value of a set of wind farms $\mathcal{W}$. And $\xi \in \mathbb{R}^{|\mathcal{W}|}$ represents the forecast errors of wind farms, $\xi \in \Xi$ is a random variable with underlying distribution Ξ. The actual output of wind farm w is $(P_{W,w}+jQ_{W,w})(1+\xi_w), \forall w \in \mathcal{W}$ and also random. In this paper, we assume the underlying distribution Ξ is unknown but fixed. We also assume that the power factor is maintained at 0.95 for any wind fluctuations.

B. A Linear Approximation

Reference [23] shows that the voltage magnitudes of PQ buses become uncertain with wind fluctuations ξ. Fig. 1 presents the voltage magnitudes with respect to wind uncertainties in a modified IEEE 24-bus system [23]. The blue

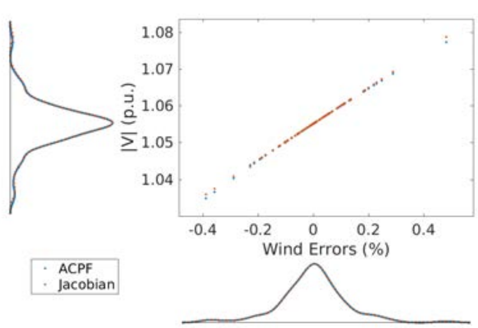

Fig. 1. Impacts of Wind Uncertainties on Voltage Magnitudes.

curve in Fig. 1 is obtained by solving a series of power flow equations, which is computationally expensive. Reference [23] proposes an approximation method using power flow Jacobian matrix to estimate the voltage magnitude changes to wind fluctuations. The red curve in Fig. 1 is calculated using the approximation method in [23]. Although the relationship between voltage magnitudes and wind fluctuation is fundamentally non-linear, Fig. 1 shows that we can get satisfying approximation using linearized power flow equations.

III. Big-M Approach to Solve CCPs

Given a two-stage chance-constrained program:

$$\min_{x,y(\omega)} c^\intercal x + F[y(\omega)] \tag{2a}$$
$$\text{s.t. } Ax \geq b \tag{2b}$$
$$\mathbb{P}_\omega\Big(G(\omega)x + L(\omega)y(\omega) \leq h(\omega)\Big) \geq 1-\epsilon \tag{2c}$$
$$x \in \mathbb{R}_+^{n_1} \times \mathbb{Z}_+^{n_2}, y(\omega) \in \mathbb{R}_+^{n_3}$$

The first stage variable x could take both continuous and integer values. Notice that the second stage variable y depends on the realization of variable ω, thus it is denoted by $y(\omega)$.

With the well-known “Big-M” approach [10]–[12], Problem (2) could be reformulated as a *deterministic* Big-M Mixed $0-1$ Integer Program:

$$\min_{x,y^k,z_k} c^\intercal x + F[y^k] \tag{3a}$$
$$\text{s.t. } Ax \geq b \tag{3b}$$
$$G(\omega^k)x + L(\omega^k)y^k - Mz_k \leq h^k \tag{3c}$$
$$\sum_{k=1}^{N} \pi^k z_k \leq \epsilon \tag{3d}$$
$$x \in \mathbb{R}_+^{n_1} \times \mathbb{Z}_+^{n_2}, y(\omega^k) \in \mathbb{R}_+^{n_3}, z^k \in \{0,1\}$$

M is a sufficiently large coefficient and N scenarios are drawn from Ω: $\omega^1, \omega^2, \cdots, \omega^N \in \Omega$. The key idea of the Big-M approach is quite simple: for scenario ω^k, if $z_k = 0$, then Eqn. (3c) becomes $G(\omega^k)x + L(\omega^k)y^k \leq h^k$; if $z_k = 1$, then Eqn. (3c) becomes $-M \leq h^k$, which is always true if M is large enough. In essence, $z_k = 0$ indicates the constraint is retained and $z_k = 1$ indicates violations are allowed for scenario ω^k. The chance constraint $\mathbb{P}_\omega(\dots) \geq 1-\epsilon$ is approximated by Eqn. (3d).

IV. Chance-constrained Optimal Reactive Power Dispatch

A. Deterministic Optimal Reactive Power Dispatch

Our previous work [23] solved a look-ahead (deterministic) optimal reactive power dispatch (LA-det-ORPD) problem with voltage security constraints. Problem (4) is a simplified version (only one snapshot) of the LA-det-ORPD problem in [23].

$$\min \quad h_B(Q_B) + h_C(Q_C) + \lambda \sum_{c=0}^{n_c} \gamma^c P_L^c \tag{4a}$$

$$\text{s.t. } P^c = A_G^c(P_G + \eta^c P_\delta^c) + A_W P_W - A_D P_D, \forall c \tag{4b}$$

$$Q^c = A_G^c Q_G^c + A_C Q_C + A_B Q_B - A_D Q_D, \forall c \tag{4c}$$

$$P_\delta^c = \mathbf{1}^\intercal (A_D P_D - A_G^c P_G - A_W P_W), \forall c \tag{4d}$$

$$P_i^c = \sum_{j=1}^{n_b} |V_i^c||V_j^c||Y_{ij}| \cos(\theta_i^c - \theta_j^c - \phi_{ij}), \forall c, i \tag{4e}$$

$$Q_i^c = \sum_{j=1}^{n_b} |V_i^c||V_j^c||Y_{ij}| \sin(\theta_i^c - \theta_j^c - \phi_{ij}), \forall c, i \tag{4f}$$

$$P_L^c = \sum_{l=1, l:i\sim j}^{n_l} g_l\big(|V_i|^2 + |V_j|^2 - 2|V_i||V_j|\cos(\theta_i - \theta_j)\big), \forall c \tag{4g}$$

$$|V^c|^- \le |V^c| \le |V^c|^+ \tag{4h}$$

$$Q_B \in \{0, Q_B^+\}, \quad Q_C^- \le Q_C \le Q_C^+ \tag{4i}$$

$$Q_G^- \le Q_G^c \le Q_G^+ \tag{4j}$$

$$i, j = 1, 2, \cdots, n_b, \quad c = 0, 1, 2, \cdots, n_c$$

The objective of Problem (4) is to minimize the operation costs of RPSDs and transmission losses while ensuring voltage security in n_c contingency scenarios. All variables with superscript ^{c} belong to contingency scenario c[1]. In this paper, we focus on the $N-1$ contingency of losing generators[2], which are modeled by the adjacency matrix of generators A_G^c. Let A_G^0 be the adjacency matrix in the normal operating condition (i.e. no contingency), A_G^c is obtained by setting the cth column of A_G^0 to zeros.

The decision variables in Problem (4) include the operating states of *discrete* RPSDs Q_B (e.g. shunt capacitors), those of *continuous* RPSDs Q_C (e.g. SVCs) and the voltage set-points of generators (i.e voltage magnitudes $|V^c|$ of PV buses). Eqn. (4e) and Eqn. (4f) are the nodal power balance constraints, P^c (Q^c) is the nodal real (reactive) power injection into the network. $A_B \in \mathbb{R}^{n_b \times n_B}$, $A_C \in \mathbb{R}^{n_b \times n_C}$, $A_D \in \mathbb{R}^{n_b \times n_D}$, $A_G^c \in \mathbb{R}^{n_b \times n_g}$ and $A_W \in \mathbb{R}^{n_b \times n_W}$ are adjacency matrices of related components. If component k is connected with bus i, then $A_\cdot(i,k) = 1$, otherwise $A_\cdot(i,k) = 0$. Alternating Current (AC) power flow equations are depicted in Eqn. (4e) and Eqn. (4f). $Y_{ij}\angle\phi_{ij} \in \mathbb{C}$ is associated with line (i,j) (from bus i to bus j) in the admittance matrix Y.

Losing generators causes significant real power imbalance P_δ^c, we adopt the *affine control* [13] scheme to proportionally allocate P_δ^c to each generator (i.e. $P_G + \eta^c P_\delta^c$). This guarantees the balance of real power after contingency [13], [23].

Eqn. (4g) calculates the real power losses and Eqn. (4h) is the *voltage security* constraints, which typically require the voltage magnitudes within desired ranges under a set of plausible contingency scenarios [24]. In this paper, we use $[0.95, 1.05]$ for normal operation analysis ($c = 0$) and $[0.9, 1.1]$ for contingency analysis ($c = 1, 2, \cdots, n_c$). Eqn. (4i) and Eqn. (4j) are the capacity constraints for RPSDs and generators.

[1]For simplicity, the normal operating condition is denoted by $c = 0$.

[2]Since transmission line failures change the system topology thus the Y matrix in Eqn. (4e) and Eqn. (4f), we could simply modify the Y matrix to be Y^c to model the cases of losing transmission lines. For simplicity, we only focus on generator contingencies in this paper.

B. Chance-constrained Optimal Reactive Power Dispatch

Motivated by the discussion in Section II, we formulate a chance-constrained Optimal Reactive Power Dispatch (cc-ORPD) problem to ensure the voltage security of the system with respect to wind uncertainties $\xi \in \Xi$ and contingencies $c \in \mathcal{C}$. The cc-ORPD problem (Problem (5)) enhances the det-ORPD problem by adding a *joint* chance constraint Eqn. (5f). The violation probability ϵ in Eqn. (5f) *explicitly* quantifies the potential risk of voltage insecurity given the joint distribution of wind and contingencies $\mathcal{C} \times \Xi$.

$$\min \quad h_B(Q_B) + h_C(Q_C) + \lambda \mathbb{E}_{\mathcal{C}\times\Xi}\big[P_L(c,\xi)\big] \tag{5a}$$

$$\text{s.t. } P = A_G(c)P_G - A_G(c)\eta(c)P_\delta^c - A_D P_D + A_W \mathrm{diag}(P_W)(1+\xi) \tag{5b}$$

$$Q = A_G(c)Q_G + A_C Q_C + A_B Q_B - A_D Q_D + A_W \mathrm{diag}(Q_W)(1+\xi) \tag{5c}$$

$$P_\delta = \mathbf{1}^\intercal A_G(c) P_G - \mathbf{1}^\intercal \overline{P_G} + P_W^\intercal \xi \tag{5d}$$

$$\text{Power Flow Equations: Eqn.(4e), (4f), (4g)} \tag{5e}$$

$$\mathbb{P}_{\mathcal{C}\times\Xi}\Big(|V(c)|^- \le |V(c,\xi)| \le |V(c)|^+ \text{ for PQ buses and } Q_G^- \le Q_G(c,\xi) \le Q_G^+\Big) \ge 1-\epsilon \tag{5f}$$

$$|V(c)|^- \le |V| \le |V(c)|^+ \text{ for PV buses} \tag{5g}$$

$$Q_B \in \{0, Q_B^+\}, \quad Q_C^- \le Q_C \le Q_C^+ \tag{5h}$$

$$i, j = 1, 2, \cdots, n_b, \quad c = 0, 1, 2, \cdots, n_c$$

The cc-ORPD problem is a two-stage chance-constrained programming problem. The *first-stage* variables are the operating states of RPSDs (Q_B and Q_C) and the voltage set points of generators (i.e. voltage magnitudes of PV buses). The *second-stage* variables include the nodal injection (P and Q), power imbalance P_δ, total line losses P_L, reactive generation Q_G, as well as the voltage magnitudes and angles of PQ buses ($|V|$ and θ). Since the parameters A_G^c and η^c depend on the contingency c, we change the notation to $A_G(c)$ and $\eta(c)$ for better understanding. Please notice that Eqn. (5b)-(5e) are equality constraints associated with random variable c and ξ, therefore the second-stage variables (e.g. P and P_L) also become random variables[3].

The cc-ORPD problem is very challenging to solve for the following three reasons: (1) some decision variables are binary, thus the feasible region of cc-ORPD is naturally non-convex; (2) the power flow equations are non-linear equations, which further increase the difficulty of solving cc-ORPD; and (3) the chance constraint Eqn. (5f) induces computationally intractable issues as discussed in Section I-B.

The third difficulty could be handled via the Big-M approach introduced in Section III. Given a set of scenarios $s^1, s^2, \cdots, s^{|\mathcal{S}|}$, where $\mathcal{S} = \mathcal{C} \times \Xi$ and each scenario $s^i = (c,\xi)^i \in \mathcal{S}$. We introduce binary variables $z^i \in \{0,1\}$ for each scenario $s^i = (c,\xi)^i$. The chance-constraint in cc-ORPD could be re-written as a set of *deterministic* inequality

[3]More rigorous notations should denote the second-state variables are functions of c and ξ (e.g. $P(c,\xi)$ and $P_L(c,\xi)$). To avoid verbose notations, we only emphasize this in the chance constraint Eqn. (5f).

constraints with binary variables z^i. Because we want to ensure the voltage security for all contingency scenarios $\mathcal{C}$, instead of drawing scenarios $(c,\xi)^i$ from $\mathcal{C}\times\Xi$, we draw samples $\xi^1,\xi^2,\cdots$ only from Ξ, and combine them with n_c contingency scenarios utilizing the fact that the generator contingency c and wind uncertainties ξ are independent. More specifically, let π_c denote the probability that contingency c happens, and ξ^k $(k=1,2,\cdots,N)$ are the wind scenarios. The cc-ORPD problem is reformulated as Problem (6), where variables with superscripts ^{c,k} are associated with contingency c and wind scenario ξ^k.

$$
\begin{aligned}
\min \quad & h_B(Q_B)+h_C(Q_C)+\lambda\sum_{c=0}^{n_c}\gamma^{c,k}\frac{1}{N}\sum_{s=1}^{N}P_L^{c,k}(P_W^s) && \text{(6a)}\\
\text{s.t. } P^{c,k} &= A_G^c P_G - A_G^c\eta^c P_\delta^{c,k} - A_D P_D \\
&\quad + A_W\text{diag}(P_W)(1+\xi^k), \forall c,k && \text{(6b)}\\
Q^{c,k} &= A_G^c Q_G^{c,k} + A_C Q_C + A_B Q_B - A_D Q_D \\
&\quad + A_W\text{diag}(Q_W)(1+\xi^k), \forall c,k && \text{(6c)}\\
P_\delta^{c,k} &= \mathbf{1}^\intercal A_G^c P_G - \mathbf{1}^\intercal\overline{P_G} + P_W^\intercal\xi^k, \forall c,k && \text{(6d)}\\
P_i^{c,k} &= \sum_{j=1}^{n_b}|V_i^{c,k}||V_j^{c,k}||Y_{ij}|\cos(\theta_i^{c,k}-\theta_j^{c,k}-\phi_{ij}), \forall c,s,i && \text{(6e)}\\
Q_i^{c,k} &= \sum_{j=1}^{n_b}|V_i^{c,k}||V_j^{c,k}||Y_{ij}|\sin(\theta_i^{c,k}-\theta_j^{c,k}-\phi_{ij}), \forall c,s,i && \text{(6f)}\\
P_L^{c,k} &= \sum_{l=1}^{n_l} g_l\big(|V_i^{c,k}|^2+|V_j^{c,k}|^2 \\
&\quad - 2|V_i^{c,k}||V_j^{c,k}|\cos(\theta_i^{c,k}-\theta_j^{c,k})\big), \forall c,k && \text{(6g)}\\
|V^{c,k}| &- M z_{c,k} \le |V^{c,k}|^+, \forall c,k && \text{(6h)}\\
|V^{c,k}| &+ M z_{c,k} \ge |V^{c,k}|^-, \forall c,k && \text{(6i)}\\
Q_G^{c,k} &- M z_{c,k} \le Q_G^+, \forall c,k && \text{(6j)}\\
Q_G^{c,k} &+ M z_{c,k} \ge Q_G^-, \forall c,k && \text{(6k)}\\
Q_B &\in \{0, Q_B^+\}, \quad Q_C^- \le Q_C \le Q_C^+ && \text{(6l)}\\
\sum_{k=1}^{N} & \frac{1}{N}\sum_{c=0}^{n_c}\pi_c z_{c,k} \le \epsilon && \text{(6m)}\\
i,j &= 1,2,\cdots,n_b, \quad c=0,1,2,\cdots,n_c, k=1,2,\cdots,N
\end{aligned}
$$

C. Linearized cc-ORPD

Problem (6) is a Mixed Integer Non-Linear Programming (MINLP) problem, which is still computationally intractable. But the major difficulty here comes from the non-linear power flow equations. As shown in Section II-B, we could obtain satisfying approximations via linearized power flow equations. Thus Eqn. (6e) and (6f) are linearized with respect to a known operating point (e.g. power flow solutions of a previous snapshot). Our future works include exploring other possible approaches to handle non-linearity of power flow equations (e.g. convex relaxation). Problem (9) is obtained by replacing Eqn. (6e)-(6f) with Eqn. (7). It is a Mixed Integer Linear Programming problem and is reliably solvable with commercial solvers.

$$
\begin{bmatrix} P-\bar{P} \\ Q-\bar{Q} \end{bmatrix} \approx \begin{bmatrix} \frac{\partial P}{\partial\theta} & \frac{\partial P}{\partial|V|} \\ \frac{\partial Q}{\partial\theta} & \frac{\partial Q}{\partial|V|} \end{bmatrix}_{\bar{P},\bar{Q},|\bar{V}|,\bar{\theta}} \times \begin{bmatrix} \theta-\bar{\theta} \\ |V|-|\bar{V}| \end{bmatrix} \quad (7)
$$

$$
P_L-\bar{P}_L \approx \begin{bmatrix} \frac{\partial P_L}{\partial\theta} & \frac{\partial P_L}{\partial|V|} \end{bmatrix}_{\bar{P},\bar{Q},|\bar{V}|,\bar{\theta}} \times \begin{bmatrix} \theta-\bar{\theta} \\ |V|-|\bar{V}| \end{bmatrix} \quad (8)
$$

$$
\begin{aligned}
\min \quad & h_B(Q_B)+h_C(Q_C)+\lambda\sum_{c=0}^{n_c}\gamma^{c,k}\sum_{s=1}^{N}P_L^{c,k}(P_W^s) && \text{(9a)}\\
\text{s.t. } & \text{Eqn. } (6b),(6c),(6d) && \text{(9b)}\\
& \text{Eqn. } (7),(8) && \text{(9c)}\\
& \text{Eqn. } (6h),(6i),(6j),(6k),(6l),(6m) \\
& \Delta|V|^- \le |V^{c,k}| - \overline{|V|} \le \Delta|V|^+ && \text{(9d)}\\
& \Delta|\theta|^- \le |\theta^{c,k}| - \overline{|\theta|} \le \Delta|\theta|^+ && \text{(9e)}\\
& i,j=1,2,\cdots,n_b, \quad c=0,1,2,\cdots,n_c
\end{aligned}
$$

V. Case Study

A. Settings

Case studies are conducted on a modified IEEE 24-bus system [23]. There are 38 contingencies considered in the case study, each one represents the scenario of losing one generator at a PV bus[4]. We assume the probability of the normal operating condition is $\pi_0 = 90\%$, and each contingency happens with equal probability, i.e. $\pi_c = 10\%/38 = 0.26\%$. By tuning the probabilities π_cs and ϵ, we could achieve a balance between a more economic system and a more secure system. The wind uncertainty ξ is assumed to be Gaussian $\xi\sim\mathcal{N}(0,5\%)$, from which 100 scenarios ξ^k are drawn and plugged in Problem (9). It is worth mentioning that solving Problem (9) solely relies on the scenarios ξ^k, it does not require any prior knowledge on the underlying distribution.

B. Simulation Results

Problem (9) was solved via Matlab2016b and Gurobi 7.5 on a Desktop with Intel i7-2600 8-core CPU@3.40GHz and 16GB RAM memory. Gurobi found the optimal solution with 0.0% gap in 330 seconds. The optimal objective value is \$1668.13. Fig. 2 demonstrates the optimal voltage set points of generators and the voltage magnitudes of PQ buses in the normal operating condition. The voltage magnitudes of bus 4 and bus 14 are fluctuating due to wind uncertainties, while some buses (e.g. bus 17, 19 and 20) remain almost the same voltage magnitudes.

Besides the optimal solution to the cc-ORPD problem, we are also interested in the actual violation probability $\hat{\epsilon}$. Let $\bar{\epsilon}$ denote the expected violation probability: $\bar{\epsilon} := \sum_{k=1}^{N}\frac{1}{N}\sum_{c=0}^{n_c}\pi_c z_{c,k}^*$, where $z_{c,k}^*$ is from the optimal solution to Problem (9). It is obvious that $\bar{\epsilon}\le\epsilon$. Let $\hat{\epsilon}$ denote the actual "out-of-sample" violation probability:

$$
\hat{\epsilon} := \sum_{k=1}^{\hat{N}}\frac{1}{\hat{N}}\sum_{c=0}^{n_c}\pi_c\mathbf{1}_{Q_G^{c,k}\notin[Q_G^-,Q_G^+] \text{ or } |V^{c,k}|\notin[|V^c|^-,|V^c|^+]} \quad (10)
$$

[4]If there is only one generator at the PV bus, losing the generator will make it to a PQ bus. For simplicity, we replace it with two generators with half capacities.

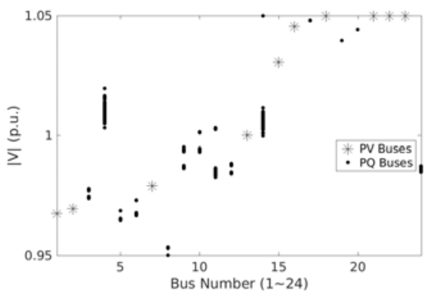

Fig. 2. Voltage Magnitudes in the Normal Operating Condition.

where $\mathbf{1}_{\text{conditions}}$ is the indicator function. We generate an independent set of $\hat{N}$ scenarios and calculate the voltage magnitudes and reactive power generations using linearized power flow equations [23] or solving the power flow equations.

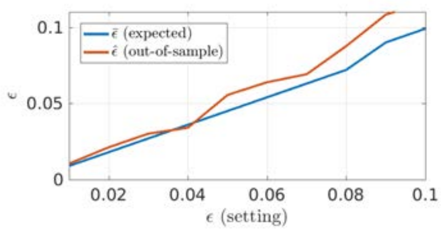

Fig. 3. Violation Probabilities.

The blue curve in Fig. 3 is the expected violation probability $\bar{\epsilon}$ from the optimal solution z^*. And the red line $\hat{\epsilon}$ is calculated on $\hat{N} = 100$ scenarios using linearized power flow equations [23]. The out-of-sample violation probability $\hat{\epsilon}$ is very close to $\bar{\epsilon}$. With a larger number of scenarios embedded in Problem (9), the expected $\bar{\epsilon}$ and actual $\hat{\epsilon}$ will be closed to the violation probability ϵ in the chance constraint.

We also compare the results of cc-ORPD (Problem (9)) with det-ORPD (Problem (4)). With a little sacrifice on the total cost, the cc-ORPD could ensures voltage security with probability 98.8%. While the results of det-ORPD lead to voltage magnitudes lower than the desired lower bound $|V^c|^-$. In the results of det-ORPD, we even observe undesirable low voltage magnitudes in the normal operating condition, which results in the large violation probability in Table I.

TABLE I
DET-ORPD VS CC-ORPD

	det-ORPD	**cc-ORPD** ($\epsilon = 0.01$)
Objective	1610.2	1668.1
$\hat{\epsilon}$	52.1%	1.2%

VI. Concluding Remarks

In this paper, we propose a chance-constrained formulation of optimal power reactive dispatch to schedule RPSDs considering uncertainties from wind and contingencies. The cc-ORPD problem is reformulated as a computationally solvable form using the Big-M approach and linearized power flow equations. Case studies demonstrate the effectiveness of the proposed cc-ORPD framework. Future works include investigating convex relaxations of power flow equations and utilizing improved versions of the Big-M approach [11], [12].

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